

**THE DISCOVERY OF THE ELECTRON
THE J. J. THOMSON EXPERIMENT**

TABLE OF CONTENTS

Historical Background	2
Pre-Lab Home Work	9
Experiment 1	11
Experiment 2	15

THE DISCOVERY OF THE ELECTRON

NOTE: The following text was taken from "From Alchemy To Quarks" by Sheldon L. Glashow, pp. 423-429.

HISTORICAL BACKGROUND

Roentgen set out to study cathode rays but was rewarded, on or about Christmas 1895, with the discovery of X-rays. Becquerel searched for a suspected but nonexistent link between Roentgen's rays and phosphorescence. Instead, he found something totally unexpected: radioactivity. In January 1896, Ernest Rutherford, a young scientist who was working at the renowned Cavendish Laboratory in Cambridge, England, wrote to his fiancée:

The Professor [his advisor, J.J. Thomson] has been very busy lately over the new method of photography discovered by Professor Roentgen. [Thomson] is trying to find out the real cause and nature of the waves, and the great object is to find the theory of matter before anyone else, for nearly every Professor in Europe is now on the warpath.

The professor and his assistant would find the parts of the atom. Thomson was about to discover the electron. Sixteen years later, Rutherford would discover the nucleus of the atom. The first of the Nobel prizes was awarded to Roentgen for his discovery of X-rays. Two years later, the Prize was split between Becquerel and the Curies.

Joseph John Thomson, J.J., as he was universally known, was director of the Cavendish Laboratories at Cambridge University from 1884 to 1918. His predecessor was Maxwell, his successor Rutherford. Thomson discovered the electron in 1897. Sixteen years later, he showed that some neon atoms weigh 10 percent more than others, thereby discovering stable isotopes.

DISCOVERY OF THE ELECTRON

Joseph John Thomson (1856-1940) discovered the first "elementary particle" in 1897. Forty years later, Thomson published his *Recollections and Reflections*. His description of his own work is given below.

The research that led to the discovery of the electron began with an attempt to explain the discrepancy between the behavior of cathode rays under magnetic and electric forces. [Hertz had shown that] magnetic forces deflect the rays in just the same ways as they would a negatively electrified particle moving in the direction of the rays.

[Perrin had shown that a metal] cylinder receives a copious negative charge when the cathode rays are deflected by a magnetic into the cylinder. This would seem to be conclusive evidence that the rays carried a charge of electricity had not Hertz found that when they were exposed to an electric force they were not deflected at all. From this he came to the conclusion that they were not charged particles. He took the view that they were flexible electric currents flowing through the ether, and that they were acted upon by magnetic forces in accordance with the laws discovered by Ampere for the forces exerted on electric currents.

Such currents would give a charge of negative electricity to bodies against which they struck. They would be deflected by a magnet. They would not be deflected by electric forces. These are just the properties that the cathode rays were for a long time thought to possess.

Cathode rays were identified as electric currents because they were deflected by magnetic fields and they caused objects in their path to become electrically charged. At the time, however, many scientists did not believe that electric currents consist of electric charges in motion. The failure of Hertz to observe the deflection of cathode rays by electric fields seemed to confirm their view. Thomson was convinced otherwise. He set out to see for himself whether cathode rays are affected by electric fields.

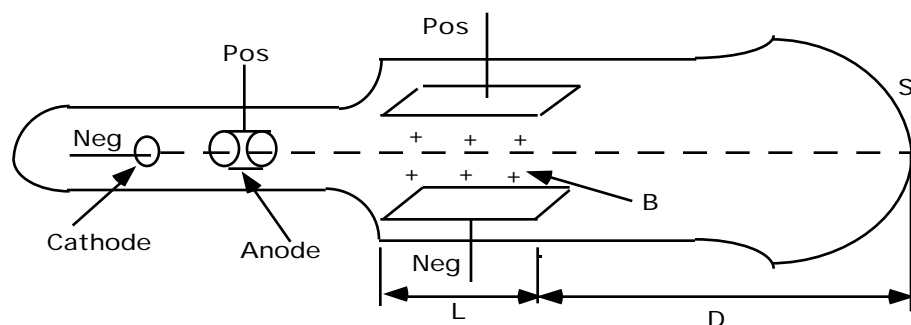


Figure 1

Thomson built the device shown in Figure 1, consisting of an evacuated glass tube with a narrow constriction. Two electrodes were inserted into one part of the tube, to which a high voltage (supplied by a Ruhmkorff coil) was applied. Cathode rays streamed off in all directions from the negative cathode. The constriction in the tube was surrounded by a metal ring that allowed a narrow beam of cathode rays to enter the larger portion of the tube. The beam struck the far end of the tube, producing a small fluorescent spot.

If cathode rays were charged particles, they must be deflected by both electric and magnetic fields. The force on a moving charge is

$$F = q (E + v \times B) \quad (1)$$

where E and B are the electric and magnetic fields at the position of the particle. However, when Thomson connected the parallel plates in his tube to a high voltage source, he saw no lasting deflection but only "a slight flicker in the beam when the electric force was first applied." Noticing that flicker made the difference between a good scientist and a great one.

Thomson realized that gas molecules in the imperfect vacuum of his tube were ionized by the electric discharge. The ions migrated to the electrodes and almost instantaneously neutralized their electric charges. "The absence of deflection is due to the presence of gas - to the pressure [in the tube] being too high," wrote Thomson, "thus the thing to do was to get a much higher vacuum. This was more easily said than done." Thomson eventually got his better vacuum, and with it he showed that cathode rays are indeed deflected by electric as well as magnetic fields. This was a necessary condition for his view of the particulate nature of cathode rays to be correct, but it was not sufficient. If cathode rays consisted of rapidly moving charged particles, then surely the properties of the particles could be measured. What was the velocity v of the electrons in the tube? What is the electric charge $-e$ carried by an electron? What is the mass m of an electron? As Thomson later wrote:

This result removed the discrepancy the effects of magnetic and electric forces on the cathode particles; it did much more than this; it provided a method of measuring v the velocity of these particles, and also e/m , where m is the mass of the particle and e is its electric charge.

MEASURING THE CHARGE - TO MASS RATIO OF ELECTRONS

Thomson subjected the cathode rays in his tube to electric and magnetic fields at the same time. Suppose the cathode rays are moving in the x -direction. The parallel plates inside the tube, when electrified, produce a known electric field E in the upward z -direction. The effect of this electric field is to drive the negatively charged cathode rays downward. An electromagnet placed outside the tube produces a known magnetic field B in the y -direction. The effect of this field is to drive the electrons upward. Suppose that both fields extend over the same length l along the trajectory of the cathode rays. The electric and magnetic forces cancel one another if the following condition is met:

$$v = E/B$$

where v is the velocity of the charge. Thomson adjusted the strength of the magnetic field so the cathode rays were not deflected by the crossed fields. From the known values of the compensating fields, he deduced the velocity of particles making up the rays.

Once he measured v in this manner, Thomson turned off the magnetic field. The glowing spot on the tube was then driven down by the electric field. From the distance the spot was deflected, Thomson determined the ratio of the charge of the electron to its mass, the quantity e/m .

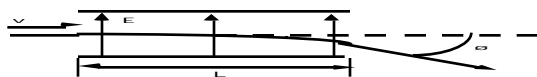


Figure 2:

An electron passes through a region in which there is an electric field E pointing up. The electron is deflected downward by a distance d .

Suppose that an electron with charge " $-e$ " and mass m is moving to the right, as shown in Figure 2. It passes through a region of length l in which there is an electric E field pointing up. If the electron is deflected downward by a distance d as it passes through the field, what is the value of e/m , its charge-to-mass ratio?

Since electrons are pushed down by an electric field pointing up, the charge of the electron is negative. The magnitude of the downward electric force is eE . The electrons accelerate down with the vertical acceleration eE/m during their traversal of the horizontal distance l . (They also fall with the acceleration of gravity g . In practice, however, g is negligible compared to eE/m , the acceleration due to the electric field.) The vertical displacement d of a uniformly accelerated body is $\frac{1}{2}at^2$,

where a is the acceleration and t is the time interval over which it is accelerated. In Thomson's experiment, $t = l/v$ and $a = eE/m$. Thus:

$$d = \frac{eEl^2}{2mv^2} \quad \text{or} \quad \frac{e}{m} = \frac{2dv^2}{El^2}$$

The unknown quantity e/m is expressed in terms of the known quantities d , E , v , and l . Notice that the deflection of the electron in this example and in Thomson's tube determines neither the value of e nor the value of m , but only their ratio.

In another experiment, Thomson measured the charge-to-mass ratio of positively charged H^+ ions and found that it was about 1000 times smaller than the corresponding ratio for electrons. It seemed plausible, from Faraday's earlier studies of electrolysis, to assume that the magnitudes of charges of the electron and the ion were the same. From this, it followed that the hydrogen atom must be about 1000 times heavier than the electron. Thomson's measurements of charge-to-mass ratios were not precise; the actual ratio of masses is closer to 1837. The essential point, however, is that Thomson showed that electrons are much lighter than atoms. Thomson pressed onward and left precision experiments for others to perform:

If e were the same as the charge of electricity carried by an atom by hydrogen - as was subsequently proved to be the case - then m , the mass of the cathode-ray particle, could not be greater than one thousandth part of the mass of an atom of hydrogen, the smallest mass hitherto recognized. These results were so surprising that it seemed more important to make a general survey of the subject than to endeavor to improve the determination of the exact value of the ratio of the mass of the electron to the mass of the hydrogen atom.

Thomson found that electrons could be produced from many different materials and in many different ways. He studied the charged particles produced when ultraviolet radiation strikes a metal surface (the photoelectric effect, to which we return in another experiment). He studies the charged particles produced when carbon or metal filaments are heated to incandescence (as in a lamp bulb).

He "measured, by methods based on similar principles to those used for cathode rays, the value of e/m for the carriers of negative electricity in those cases, and found that it was the same as for cathode rays."

Although they were discovered in electric discharges, Thomson showed that his new particles were everywhere; electrons are parts of the atoms that make up matter. Thomson saw no escape from the following conclusions:

That atoms are not indivisible, for negatively electrified particles can be torn from them by the action of electric forces, impact of rapidly moving atoms, ultraviolet light or heat.

That these particles are all of the same mass, and carry the same charge of negative electricity from whatever kind of atom they may be derived, and are constituents of all atoms.

That the mass of these particles is less than one thousandth part of the mass of a hydrogen atom.

I at first called these particles corpuscles, but they are now called by the more appropriate name electrons. I made the first announcement of the existence of these corpuscles at a Friday Evening Discourse at the Royal Institute on April 29, 1897. There were very few people who believed in the existence of these bodies smaller than atoms. I was even told afterward by a distinguished physicist that he thought I had been pulling their legs. I was not surprised at this, as I myself had come to this explanation of my experiments with great reluctance, and it was only when I was convinced that the experiment left no escape from it that I published my belief in the existence of bodies smaller than atoms.

Thomson knew what the charge of an electron should be. Faraday's studies of electrolysis in the 1830s showed that the charge carried by a singly ionized atom is

$$e = \frac{96.500C}{NA} \quad 1.6 \times 10^{-19} \text{ coulombs} \quad (2)$$

where NA is Avogadro's number. Thomson believed (correctly) that an ion is an atom that has lost or acquired one or more electrons, so that the electron has a charge given by Equation 2. To confirm his suspicion, Thomson set out to measure directly the charge on an electron. His experimental technique involved the physics of fogs and clouds.

As the warm moist air of a summer day rises, its pressure decreases and water vapor condenses about dust particles or ions to form droplets. A billowing cloud is formed. Taking his lead from this phenomenon, Thomson passed X-rays through damp and dust-free air in a flask to produce a large number of singly charged ions. When the pressure in the flask was reduced, water vapor condensed about each ion, forming a mist of charged water droplets. The number of droplets was found by studying the appearance of the mist. From the number of droplets and the measured electric conductivity of the mist, Thomson determined the charge on each droplet.

This difficult experiment was more easily said than done. Thomson did not obtain as precise a determination of e as he had hoped for. His published results ranged from two-thirds to twice the expected value given by equation 2. Thomson's methods were later refined by his younger colleague C.T.R. Wilson. Years later, Wilson invented the *cloud chamber*, an instrument by which rapidly moving charged particles can be "seen" as they leave visible trails of water droplets along their paths. We shall hear more about cloud chambers later on.

In 1906, the American physicist Robert Andrews Millikan, "being dissatisfied with the variability" of Thomson's results, repeated the experiment, "without obtaining any greater consistence." His failure made his try harder:

This attempt, while not successful in the form in which it had been planned, led to a modification of the cloud method which seemed at the time, and which has actually proved since, to be of far-reaching importance. It made it for the first time possible to make all the measurements on individual droplets, and thus not merely to eliminate all of the questionable assumptions and experimental uncertainties involved in the cloud method of determining e,

but, more important still, it made it possible to examine the properties of individual isolated electrons and to determine whether different ions actually carry one and the same charge. That is to say, it now became possible to determine whether electricity in gases and solutions is actually built up of electrical atoms, each of which has exactly the same value.

Millikan finally got a definitive result for the electron's charge by a straightforward procedure known to later generations of physics students as the Millikan oil drop experiment. This will be the second experiment that will be carried out in this Nobel Experiments Laboratory.

THE REAL J.J. THOMSON TUBE EXPERIMENT

We are now ready to consider how Thomson measured the ratio of charge to mass, q/m , for what he called "cathode corpuscles." His apparatus (Figure 1) consisted of a highly evacuated glass tube into which several metal electrodes were sealed. Electrode C is the cathode from which the rays emerged. Electrode A is the anode, which was maintained at a high positive potential so that a discharge of cathode rays passed to it. A narrow beam of the rays passed into the region between the two plates P and P'. After passing between the plates, the rays struck the end of the tube, where they caused fluorescent material at S to glow.

The deflection plates P and P' were separated a known amount, so that when they were at a known difference of potential the electric field between them could be computed. We shall assume that the field was uniform for a distance L between the plates and zero outside them. When the upper plate P was made positive, the electric field deflected the negative cathode rays upward.

The trajectory of the cathode corpuscle is obtained in the same way that trajectories were found for projectiles in the gravitational field of force in mechanics. The only difference is that here the constant electric force is upward and is limited to the region between the plates. The gravitational force mg and the electrostatic force between two corpuscles are so small that they may be neglected.

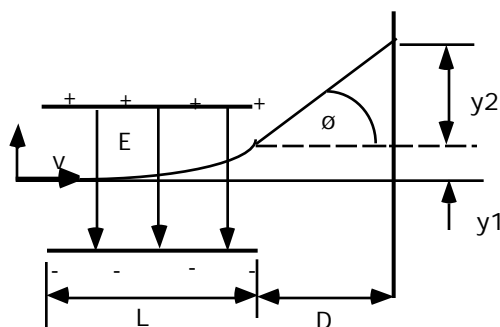


Figure 3: Electrostatic deflection of cathode rays.

In Figure 3, the cathode rays enter the region between the plates at the origin 0 with a velocity v_x , this velocity will continue to be the horizontal component of the velocity of the rays since there are no forces in the x -direction. Between the plates, however, the rays experience an upward acceleration from the constant electric field E . Eliminating the time between the equations of motion in the x - and y -directions yields the equation for the parabolic trajectory between the plates:

$$y = \frac{qEx^2}{2mv_x^2} \quad (3)$$

Beyond the plates, the trajectory is a straight line because the charge is then moving in a field-free space. The total deflection of the beam is sum of the deflections in regions y_1 and y_2 and can be shown to be:

$$y = \frac{qEL}{2mv_x} \left(\frac{L}{2} + D \right) \quad (4)$$

If q/m is regarded as a single unknown, then there are two unknowns in this equation. The initial velocity of the rays, v_x , must be determined before q/m can be found. We need another equation involving the initial velocity v_x , so that this unknown velocity can be eliminated between the new equation and Eq. 4.

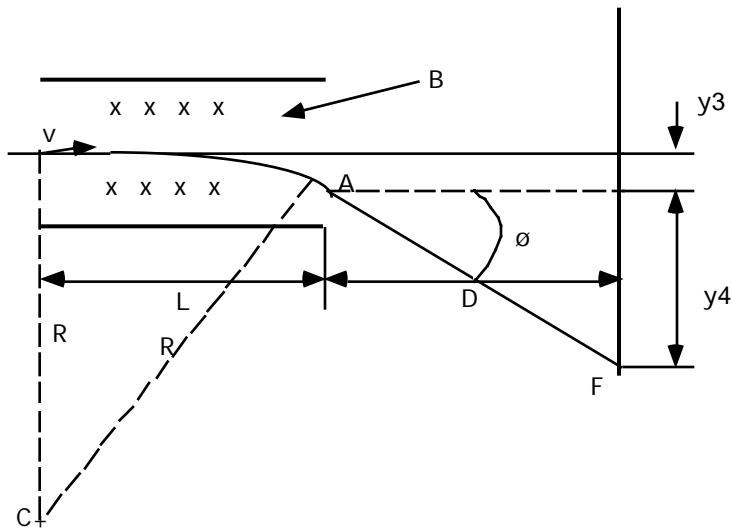


Figure 4: Magnetic deflection of cathode particles.

Thomson obtained another equation by applying a magnetic field perpendicular to both the cathode corpuscle beam and the electric field. It is represented in Fig. 4 as being into the page and uniform everywhere within the x-marked area. Thus the electric and magnetic forces acted on the cathode rays in the same geometric space.

Figure 4 shows the situation when the magnetic field alone is present. The negatively charged rays experience a force that is initially downwards. This force is not constant in direction but is always normal to both the field and the direction of motion of the rays. Therefore the cathode corpuscles move in a circular path. The center of curvature of the trajectory is at C, and the radius of curvature of the path is

$$R = mv_x/qB,$$

where v_x is the initial velocity of the rays in the x-direction. Referred to the origin O, the equation of this circular path is

$$x^2 + (R + y)^2 = R^2.$$

Thomson's procedure for determining v_x was the simultaneous application of the electric and the magnetic fields. If these are adjusted so that there is no deflection on the screen, then the force of the electric field on the charged particle is balanced by that of the magnetic field. For this condition of balance, we found that

$$F = qE - qv_xB = 0,$$

or in terms of v_x ,

$$v_x = \frac{E}{B} . \quad (5)$$

For this particular ratio of the fields, the particle goes straight through both fields. It is undeflected, and therefore the measurement of v_x does not depend on the geometry of the tube. The velocity thus determined may be substituted into Eq. (4).

Thomson measures q/m for cathode rays and found a unique value for this quantity that was independent of the cathode material and the residual gas in the tube. This independence indicated that cathode corpuscles are a common constituent of all matter. The modern accepted value of q/m is $(1.758796 \pm 0.000010) \times 10^{11}$ coulombs per kilogram. Thus Thomson is credited with the discovery of the first subatomic particle, the electron. Because it was shown later that electrons have a unique charge e , the quantity he measured is now denoted by e/me . He also found that the velocity of the electrons in the beam was about one-tenth the velocity of light, much larger than any previously measured material particle velocity.

The velocity of the electrons along the x -direction could also have been determined by using the conservation of energy without the applied magnetic field. Thomson could have measured the potential difference between the cathode and the anode and obtained the electron velocity v_x :

$$v_x^2 = \frac{2qV}{m} .$$

We will calculate e/m using both of these methods.

e/m of the Electron
J.J. Thomson Experiment

Pre-Lab Home Work

The following questions and numerical problems have been designed to introduce the concepts and theories that will be used in the laboratory experiment.

These problems are to be worked, the solutions presented in a neat logical manner, and then handed in at the beginning of the laboratory period.

1. An electron moving horizontally in a vertical plane with a speed of 5.0×10^7 m/s enters a region where there is a uniform electric field of 20 v/cm directed upward. Find the electron's coordinates referred to in the point of entry and the direction of its motion at a time 4×10^{-8} s later.
2. The dimensions of a typical cathode-ray tube are: the distance between the deflection plates is 0.50 cm and their length is 1.60 cm. The distance from the end of the plates to the screen is 15.0 cm.
 - a. If the electrons start from rest at the cathode, what is their velocity as they enter the deflection plates for an accelerating voltage of 1136 V between the anode and cathode?
 - b. If the potential difference between the deflection plates is 50 V, find the coordinates and direction of motion of the electrons and the end of the deflecting plates $x = L$.
 - c. Determine the total deflection on the screen.
 - d. What would be the magnitude and direction of a magnetic field that would cause the particles to go between the plates undeflected?
3. Two deflected plates are separated by 1.0 cm and have a potential difference of 4,000 V applied. A uniform magnetic field of flux density 0.02 Tesla is applied perpendicular to the deflected plates so that an electron moving perpendicular to both fields experiences no net force.
 - a. Show in a diagram the relative orientation of the electric field vector, the magnetic induction vector, and the velocity of the electron.
 - b. Calculate the speed of the electron.
 - c. What is the radius of the electron orbit when the electric field is removed?
4. A uniform magnetic field whose induction is 4×10^{-4} Tesla acts on the electron beam over the distance L in the cathode-ray tube in Problem 2. The field is normal to the plane of the trajectory and directed outward.
 - a. Find the y -coordinate and the direction of motion of the electron when $x = L$.
 - b. What is the total deflection on the screen S ?
 - c. What would be the deflection on the screen computed both from the approximate and the exact expressions for the radius of curvature if the magnetic induction extended the whole distance, $L + D$, from O to the screen?

5. The cathode-ray tube of Problem 2 is mounted horizontally and oriented so that the screen faces north. Given that the earth's magnetic induction is 5×10^{-5} Tesla at a dip of 53° , find the magnitude and direction of the horizontal component and of the vertical component of the displacement of the electron beam with respect to the original position on the screen when the tube is turned 90° so that it faces west. Assume that the earth's field extends only over the distance $L + D$ and that the deflections are small.
6. In a cathode-ray tube of the form shown in Problem 2, electrons are accelerated from rest through a potential difference V . These then enter a vertical electric field E that extends over the distance L , and also a magnetic field having flux density B normal to the paper over the distance $L + D$. If both fields are adjusted so that there is no deflection on the screen, show that

$$\frac{e}{mc} = \frac{E^2 L^2 (L + 2D)^2}{2VB^2 (L + D)^4}.$$

7. Derive the expression (6a) in the first experiment.

EXPERIMENT 1

e/m for the electron using the J.J. Thomson Method

1. INTRODUCTION

A stream of electrons can be deflected easily by electrostatic and electromagnetic fields. In 1897, J.J. Thomson used this fact in devising an experiment to determine the physical properties of the electron. The Electron Beam Deflection Tube, CENCO Catalog #32054, uses the same principle for the determination of e/m by the original Thomson method. Other uses include:

- Confirmation of the equation $F = eE$
- Verification of the Lorentz equation $F = ev \times H$
- Demonstration of magnetic focusing and (at high magnetic fields) the "electron mirror"

2. DESCRIPTION

The apparatus consists of a stand, a pair of Helmholtz coils, and the deflection tube. This clear glass bulb contains a "gun" which emits a narrow ribbon of cathode rays. These rays are intercepted by a flat mica sheet, one side of which is coated with a luminescent screen. The other side of the mica sheet is printed with a centimeter graticule. The luminescent screen reacts when electrons strike it, making it possible to see the path followed by the electrons. The mica sheet is held at 15° to the axis of the tube by two deflecting plates, each connected to a 4 mm plug mounted on a plastic cap. The "hairpin" filament of the electron gun is connected to 4 mm sockets in a plastic cap at the end of the neck; connections to the anode of the gun are made by a 4 mm plug mounted on the side of the neck.

The #31307 Universal Stand is included for supporting the tube. Gray plastic jaws, 16 cm long and 14 cm apart, hold the sides of the bulb securely. These jaws may be rotated 360° around their horizontal axis. They are constructed of a durable, heat-resistant material that provides high electrical insulation in the regions where power supplies are connected to the apparatus.

The jaws are attached to a sturdy blue enameled stand, made of a light alloy. The base of the stand also has sockets for mounting the Helmholtz coils (#31308) supplied with the apparatus. These coils create a uniform magnetic field within the experimental area. Each coil has 320 turns of 22 gauge enameled copper wire wound on a plastic form, 13.6 cm in diameter. The coils are mounted on stainless steel support rods.

3. SPECIFICATIONS

Maximum filament voltage	7.5 V
Anode voltage	1500-5000 V
Typical operation	2000 - 4500 V
Anode current	1 mA

4. ASSEMBLY

CAUTION: High voltage is used to power this apparatus. Complete all assembly steps before plugging in or turning on the power supply. It is also important to observe all cautions listed with your power supply.

To mount the tube in the stand, begin by squeezing the jaws. With slight pressure, the jaws will open enough to allow insertion of the tube. When the tube is securely in place, release the pressure on the jaws and slide the jaw clamps forward, locking the tube in position.

The Helmholtz coils must also be assembled into the stand. They can be mounted easily in the holes in the base using the plastic sleeve that slides along the rod. Two 4 mm sockets are located on the side of each coil. Connect the power supply to each of the sockets marked A, with the z sockets interconnected.

5. OPERATION

Electromagnetic Deflection: Connect the apparatus to the #31392 5 kV EHT Power Supply as shown in the circuit diagram, Figure (1). Both deflecting plates should be at anode potential. Before turning on the power supply, set the voltage to zero. Switch on the power supply and adjust the voltage until a bright beam appears. Observe the path of the undeflected beam.

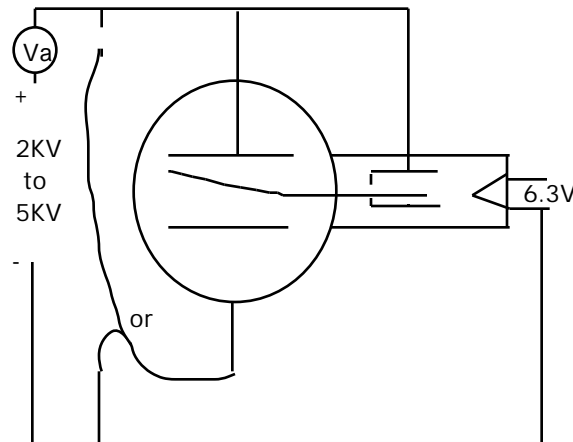


Figure (1): Circuit diagram

Energize the Helmholtz coils and observe (with reference to the screen) that:

- with V_a fixed, the radius of the beam path decreases with increase in the coil current I_B
- with I_B fixed, the radius increases with increase in anode potential V_a , indicating a high velocity
- the path of the luminous beam is circular, the deflection being in a plane perpendicular to that of the electromagnetic field.

An electron of mass m and charge e moving at right angles to a magnetic field B will experience a central force Bev constraining it to a circular path, in accordance with the equation

$$Bev = \frac{mv^2}{r} \quad (1)$$

where v is the velocity of the electron and r is the radius of curvature.

Electrostatic Deflection: Disconnect the Helmholtz coils and establish a potential difference V_p across the two deflecting plates with one plate at cathode potential. Observe that the beam is deflected by and in the plane of the electrostatic field E , following a curved path which is not circular.

The path can be shown to be a parabola governed by the equation

$$y = 1/2 (e/m)E \frac{x^2}{V} \quad (2)$$

where y is the linear deflection achieved over a linear distance x .

Equations (1) and (2) contain the three unknowns e , m , and v . If e and m are combined to give the specific charge, the equations can be solved to evaluate e/m and v .

Determination of the ratio e/m by deflection: Thomson showed that if an electrical field of strength E is applied at the same time as, and perpendicular to, an electromagnetic field B' , so that the two deflections are in the same plane but in opposite directions, these can be balanced by adjustment of the fields so that

$$Ee = B'ev \quad (3)$$

The velocity is thus found to be

$$v = E/B' \quad (4)$$

Connect the electrical circuit as before and establish the condition of balance by varying the current I_B in the Helmholtz coils with a fixed potential difference V_P between the plates (see notes on experimental results, below). Then

$$E = \frac{V_P}{d} \quad \text{and} \quad B' = I_B k \quad (5)$$

where d is the spacing between the plates and k is a constant. Substitution for v in equation (1) gives a measure of e/m . It will be seen that for a given velocity, the product of the magnetic flux B and the radius r of the resulting path is constant. Therefore, while it is possible to measure any r' corresponding to a flux B' , it is more convenient to select a constant circle of radius r and measure the flux B necessary to produce it. Equations (1) and (4) can be combined in terms of measurable quantities so that

$$e/m = \frac{V_P}{BB' dr} = \frac{V_P}{I' k^2 dr} \quad (6)$$

For circles passing through the origin (which is at the exit aperture of the anode) and the points

$$(x, \pm y), r = \frac{x^2 + y^2}{2y} \quad (6a)$$

NOTE: In some tubes, the origin of the graticule is not at the anode. If this is the case, a correction to the radius should be made.

RESULTS:

In general, the values of e/m obtained by this method are in error by a factor of about 2. An investigation of the electric field strength can be made using equation (2). From the parabola obtained and using the accepted value of e/m , the field strength can be shown to be only about 50% of the theoretical value. This is due to the geometry of the deflecting plates.

Determination of e/m by magnetic deflection: More precise measurements of e/m can be made by assuming that the velocity of the electrons is governed by the "gun" equation,

$$eV_a = (1/2)mv^2 \quad (7)$$

together with measurements made when the electrons are deflected by the magnetic field alone. Combining equations (7) and (1),

$$e/m = \frac{2V_a}{B^2 r^2} \quad (8)$$

TYPICAL RESULTS

B adjusted for circles to pass through points (10, ±2.6)

V_a	I_B (y+)	I_B (y-)	I_B mean	$e/m \times 10^{11}$	$v \times 10^7$
3000 V	0.210 A	0.176 A	0.193 A	1.93 coul/kg	3.40 m/s
4000	0.245	0.210	0.227	1.86	3.85
4500	0.270	0.225	0.247	1.77	3.98

Mean value for e/m - 1.85×10^{11} coulomb/kg

Specific charge of electron = 1.7589×10^{11} coulomb/kg

Measurement of e and m: Since the mass of the electron cannot be measured from its gravitational effect, a measure of the electronic charge e must be made by, for example, Millikan's method. The mass quantity m can then be deduced from the measurement of e/m.

CALCULATION OF MAGNETIC FIELD

The Helmholtz arrangement presents a substantially uniform field in the central region of the coils calculated as follows:

$$\text{Field } H = \frac{8n}{5\sqrt{5}} \cdot \frac{I}{r} \quad \text{amp/metre}$$

where n = no. of turns (320), r = mean radius (0.068 m), I = current in amps.

$$\text{Flux density } B = \frac{32}{5\sqrt{5}} \frac{n}{r} \cdot \frac{I}{r} \cdot 10^{-7} \quad \text{weber/sq. meter}$$

To obtain a continuous field of about 30 amp/m, an input of 12 V and about 1.0 A should not be exceeded. The maximum short-duration field of 45 amp/m from an input of 18 V and 1.5 A should not be applied for more than 10 minutes.

EXPERIMENT 2

e/m for the electron using the K.T. Bainbridge Tube

The e/m Vacuum Tube has been designed for determining the ratio of charge to mass of an electron. It may also be used to demonstrate the curving of a beam of charged particles as they pass through a magnetic field, and as an aid in explaining the principle of the mass spectrometer. The Helmholtz Coils have been designed large and rigid to provide the uniform magnetic field required in the operation of the tube. The axis of the coils may be inclined to the dip angle of quantitative measurements and to a horizontal position of classroom demonstrations.

The tube is essentially the same as one described by K.T. Brainbridge (*American Physics Teacher* **6**, 35, 1938) who states that "Historically the method is of great interest as, in principle, it is the same as that described by A. Schuster in 1890 (*Proc. Roy. Soc.* **47**, 526, 1890) and used by W. Kaufman in 1897 (*Wied. Ann.* **61**, 544, 1897)." The present apparatus is based upon an improved version built and extensively tested by professor Ralph P. Winch, Williams College, and incorporates various additional improvements developed in our own laboratory.

The beam of electrons in the tube is produced by an electron gun composed of a straight filament surrounded by a coaxial anode containing a single axial slit. See Fig. 1a and 1b for schematic diagrams of the tube. Electrons emitted from the heated filament F are accelerated by the potential difference applied between F and the anode C. Some of the electrons come out as a narrow beam through the slit S in the side of C. When electrons of sufficiently high kinetic energy (10.4 electron volts or more) collide with mercury atoms, mercury vapor being present in the tube, a fraction of the atoms will be ionized. On recombination of these ions with stray electrons the mercury-arc spectrum is emitted with its characteristic blue color. Since recombination with emission of light occurs very near the point where ionization took place, the path of the beam of electrons is visible as the electrons travel through the mercury vapor.

The magnetic field of the Helmholtz Coils causes the stream of electrons to move in a circular path the radius of which decreases as the magnetic field increases. By proper control of the magnetic field the sharp outer edge of the beam can be made to coincide with any one of five bars spaced at known distances from the filament.

Each coil of the pair of Helmholtz Coil has 72 turns of copper wire with a resistance of approximately one ohm. The approximate mean radius of the coil is 33 centimeters. The coils are supported in a frame that can be adjusted with reference to the dip angle so that their magnetic field will be parallel to the earth's magnetic field but oppositely directed. A graduated scale indicates the angle of tilt. A safety device prevents the coils from accidentally dropping back to the horizontal. Wooden seats with retaining straps cradle the tube in a central plane midway between the two coils.

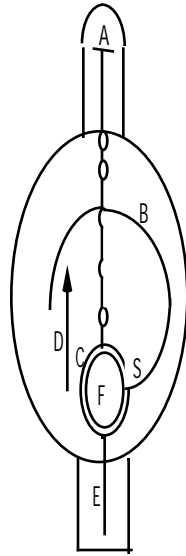


Figure 1

Fig. 1a is a sectional view of the tube and filament assembly.

- A. Five cross bars attached to staff wire.
- B. Typical path of beam of electrons
- C. Cylindrical anode.
- D. Distance from filament to far side of each of the cross bars.
- E. Lead wire and support for anode.
- F. Filament.
- G. Lead wires and supports for filament.
- L. Insulating plugs.
- S. Slit in cylindrical anode.

THEORY.

When a charged particle such as an electron moves in a magnetic field in a direction at right angles to the field it is acted on by a force, the value of which is given by

$$F = Bev \quad (1)$$

where B is the magnetic-flux density in webers/meter², e is the charge on the electron in coulombs, and v is the velocity of the electron in meters/sec. This force causes the particle to move in a circle in a plane perpendicular to the magnetic field. The radius of this circle is such that the required centripetal force is furnished by the force exerted on the particle by the magnetic field. Therefore

$$\frac{mv^2}{r} = Bev \quad (2)$$

where m is the mass of the electron in kilograms, and r is the radius of circle in meters, i.e. D/2. If the velocity of the electron is due to its being accelerated through a potential difference V, it has, due to its velocity, a kinetic energy of

$$\frac{1}{2} mv^2 = eV \quad (3)$$

where V is the accelerating potential in volts. Substituting the value of v from Eq. (3) into Eq. (2)

$$\frac{e}{m} = \frac{2V}{B^2 r^2} \quad (4)$$

Thus when the accelerating potential, the flux density of the magnetic field, and the radius of the circular path described by the electron beam are known, the value of e/m can be computed, and is given in coulombs/kg by Eq. (4) if V is in volts, B is in webers/m², and r is in meters. **NOTE:** Eq. (4) will give e/m in the electromagnetic units of abcoulombs/gm if V is in abvolts, B is in gauss, and r is in cm.

The magnetic field which causes the electron beam to move in a circular path has the magnetic-flux density B (in webers/m²) which, in terms of current through the Helmholtz coils and certain constants of the coil, is

$$B = \frac{8\mu_0 NI}{\sqrt{125}a} \quad (5)$$

where

N is the number of turns of wire on each coil,
 I is the current through coils in amperes,
 a is the mean radius of coil in meters,
 μ_0 is the permeability of empty space, which is
 4×10^{-7} weber/ampere meter.

ACCESSORY APPARATUS

The following apparatus will be needed to perform this experiment. Most of it, or the equivalent, will normally be already available. See circuit diagrams, Fig. 2 and Fig. 3.

Filament Circuit

Ammeter, 10A, D.C. 2%
 Rheostat, Slide-Wire, 2.5 ohms, 13A
 Power Supply, 0-15V, 5A D.C.
 or
 Storage Battery, 6 volts, 107 ampere-hours

Accelerating Circuit

Milliammeter, 25mA D.C. 2%
 Voltmeter, Type A, 30/15/3V D.C., 0.5%
 Power Supply, 0-300V, 30mA D.C.

Field Circuit

Ammeter, Type A, 5A D.C., 0.5%
 Rheostat, Slide-Wire, 5 ohms, 9.2A
 Rheostat, Slide-Wire, 170 ohms, 1.6A
 Power Supply, 0-15V, 5A D.C.
 or
 Storage Battery, 12 volts, 60 ampere-hours.

In addition, three sp/st knife-type switches are desirable. A magnetic dip needle will also be needed.

SETTING UP THE APPARATUS

The Helmholtz Coil Assembly is partially "knocked down" for shipment. The parts removed are a long and graduated strip attached to a flush plate, a right-angle casting with clamp, and a rubber bumper. The rubber bumper is to be located on the base board so it will be beneath the bumper on the hinged board. The flush plate is attached to the base board by the three screws that are provided. The right-angle casting with clamp is attached to the under side of the hinged board. This is to be used as a friction brake and should be so regulated that if the clamp screw is loosened while the e/m tube is mounted, the hinged board will settle slowly without shock.

The room should be fairly well darkened for demonstrating and making measurements with the e/m tube. If a table about 16 inches high is available, its use will permit the observer to lean over the Helmholtz Coils and view the beam perpendicular to its plane. If the coils are placed on a table of normal height the observer will have to stand on a chair to see the beam from the proper direction. The wooden box used for shipping the coils is of the right height to make a good support on which to place the apparatus. Its use is suggested if a table or other support is not available.

Place the tube in its support in the center of the frame. The end of the tube from which the leads extend should be adjacent to that part of the frame on which connectors are mounted. Secure the tube in place with the straps.

Orient the Helmholtz Coils so that the e/m tube will have its long axis in a magnetic north-south direction, as determined by a compass. Measure the magnetic inclination at this location with a good dip needle. Tip the coils up until the plane of the coils makes an angle with the horizontal equal to the complement of the dip angle. The axis of the coils should now be parallel to the earth's magnetic field.

Make electrical connections to the tube as shown in Fig. 2. Use a power supply with variable D.C. output or a 6-volt storage battery to supply the filament current. See Supplementary Comments for suggestions relative to the use of an A.C. filament supply. Use a slide-wire rheostat of 2.6 ohms resistance to control the filament current. An ammeter of 5- or 10-ampere range and 2% accuracy is satisfactory for measuring this current. Use a high-resistance voltmeter of 1000 ohms per volt, or higher, to measure the accelerating voltage accurately. A voltmeter of 0.5% accuracy, or better, is essential here.

Make electrical connections to the Helmholtz Coils as shown in Fig. 3. The ammeter used in this circuit for measuring the field current should have 0.5% accuracy or better. The two coils are connected in series and the current sent through them in such a direction that their magnetic field is oppositely directed to the earth's field. Knife-type switches in each of the three circuits, as shown, will be helpful. Tape any bare connections at the tube to avoid short circuits.

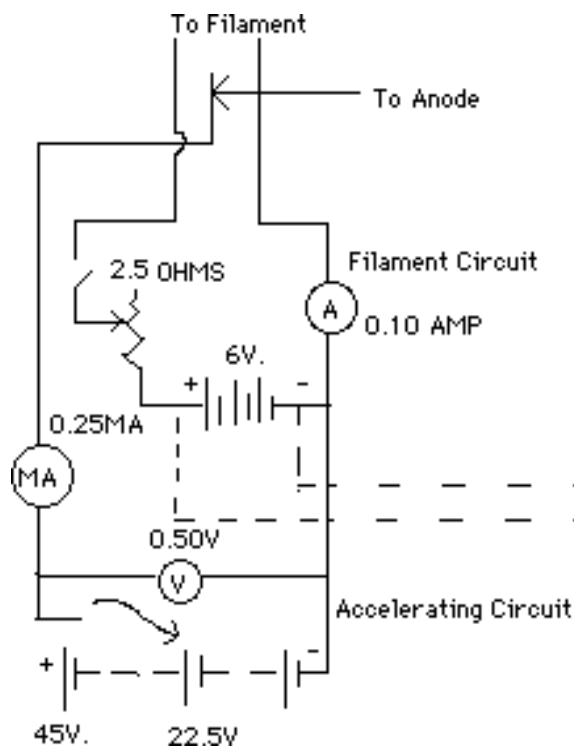


Figure 2

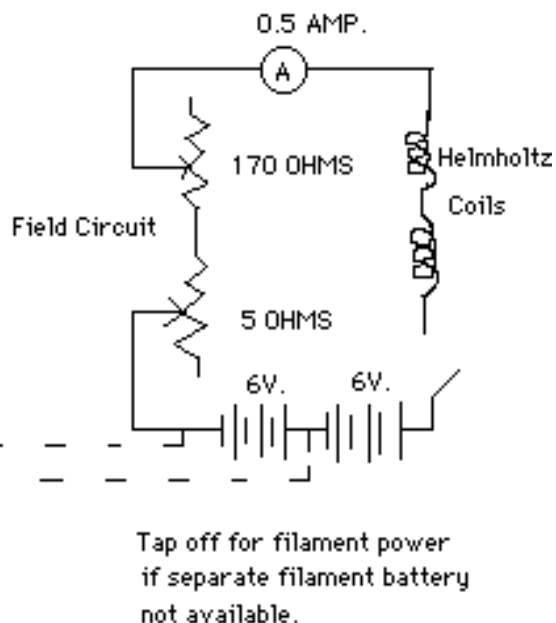


Figure 3

OPERATION

The filament of the e/m tube has the proper electron emission when carrying 2.5 to 4.5 amperes. Exceeding 4.5 amperes will materially shorten the tube's life. Good results can be obtained using an accelerating voltage of 22 1/2 to 45 volts. An electron emission current of 5 to 10 mA gives a visible beam. For maximum filament life operate the tube with the minimum filament current necessary for proper electron emission. It is helpful and good practice to apply about 25 volts accelerating potential to the anode before heating the filament. Always start with a low filament current and carefully increase it until the proper electron emission is obtained. Since the electron beam appears suddenly it is advisable to increase the filament current slowly to avoid overheating and possibly burning out the filament.

Apply 20 to 30 volts accelerating potential to the tube. With maximum resistance in the filament circuit, close the switch in this circuit. Gradually decrease this resistance until the milliammeter indicates 5 to 10 mA and the electron beam strikes the tube wall. Rotate the tube in its cradle until the electron beam is horizontal and the bars extend upward from the staff wire. Note that the electron beam is deflected slightly toward the base of the tube by the earth's magnetic field.

Now send a small current through the Helmholtz Coils. If the magnetic field of the coils straightens the beam or curves it in the opposite direction, the coils are correctly connected. If this field increases the deflection toward the base, the current is flowing through the coils in the wrong direction and the connections must be reversed. If no effect is observed on the beam, the field due to one coil is opposing that of the other and connections to one coil must be reversed. Make any necessary changes and increase the field current sufficiently to cause the beam to describe a circular path without striking the wall of the tube. The apparatus should now be ready for obtaining the necessary data to compute the value of e/m.

DATA AND RESULTS

In the manufacture of the e/m tubes the crossbars have been attached to the staff wire and this assembly attached to the filament assembly in precise fixtures which accurately control the distances between the several crossbars and the filament. Distances given are to the far side of the crossbar.

Crossbar Number	Distance to Filament (D)
1	0.065 meter
2	.078 meter
3	.090 meter
4	.103 meter
5	.115 meter

These distances are the diameters of the several circles that the electron beam will be caused to describe. Following the prescribed procedure, heat the filament until the electron beam is visible. Allow a few minutes for warm-up and temperature equilibrium. Following the prescribed procedure, heat the filament until the electron beam is visible. Allow a few minutes for warm-up and temperature equilibrium.

Close the circuit comprising the Helmholtz Coils and adjust the value of the current until the electron beam is straight. One method is to adjust the beam until it is aligned with a straightedge held as close to the tube as possible. Another method is as follows. Increase the current through the coils until the beam shows a slight curvature as compared with the straightedge and record the value of the current. Then decrease the current until the beam shows an equal curvature in the opposite direction and record the current. The average of these values will be the value of the current required to straighten the beam. This adjustment should be made carefully and accurately. When the beam is straight the magnetic field of the coils just equals the earth's magnetic field. Record the value of this current as I_1 in a table similar to Table 1. The value of I_1 is dependent only on the accelerating voltage, V , and the earth's magnetic field. It is constant unless V is changed; in which case I_1 must be redetermined.

Increase the field current until the electron beam describes a circle. Adjust its value until the sharp outside edge of the beam strikes the outside edge of a crossbar. Record this value as I_2 in the table. The outside edge of the beam is used because it is determined by the electrons with the greatest velocity. The electrons leaving the negative end of the filament fall through the greatest potential difference between filament and anode, and have the greatest velocity. It is this potential difference which the voltmeter measures.

Determine and record the field current required to cause the electron beam to strike each of the other bars. Subtract I_1 from each value of I_2 and record I . This is the value of the current to be used in Eq. (6) for computing e/m. Compute e/m for each value of I .

Repeat the above and determine e/m using a different value of accelerating voltage. Compare the results with the accepted value of e/m, which is 1.76×10^{11} coulomb/kg.