

Name \_\_\_\_\_

**Exam Three**

**CS139 - Theory of Computing  
Drake University - Spring, 2004**

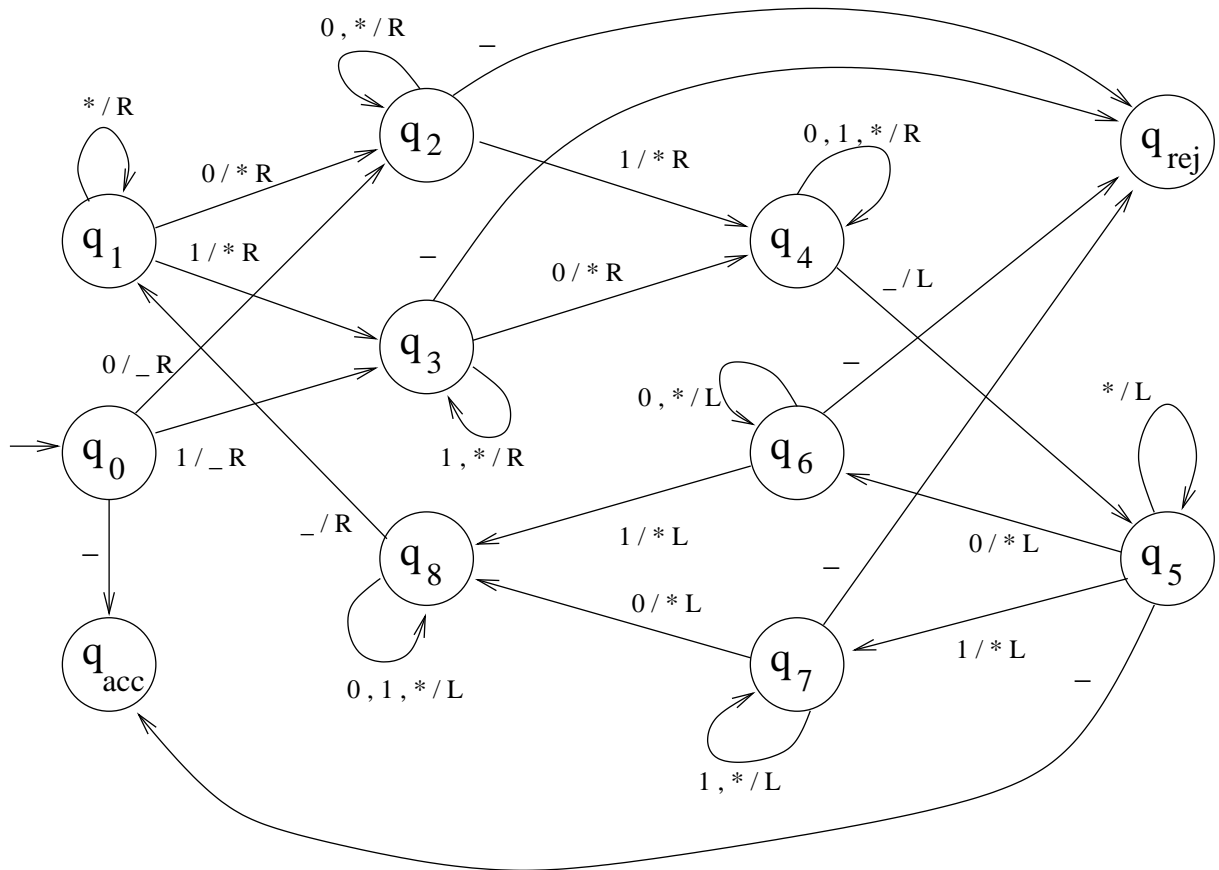
*Directions: Do all five problems. They have the same weight. Show all work. Please work first on problems with which you are more comfortable.*

**Problem 1.**

(a) Given a Turing Machine  $M$ , is it always possible to construct another Turing Machine  $M'$  that has the same language as  $M$ , but which never enters its “reject” state (even if  $M$  would)? Explain.

(b) Given a Turing Machine  $M$ , is it always possible to construct another Turing Machine  $M'$  that has the same language as  $M$ , but which eventually halts, no matter what input string it is given? Explain.

**Problem 2.** Consider the following diagram that details the behavior of a certain deterministic Turing Machine.



Using configuration notation, show all of the steps involved in processing the binary input string 001101 until the Turing Machine halts.



**Problem 3.**

Explain in detail how to design a Turing Machine that takes a binary string as input and reading this left to right checks that each time the each block of identical input symbols is longer than the previous block. So for example, it would accept the string 001110000001111111 since the blocks of identical symbols have lengths 2, 3, 6, 7, and this sequence is strictly increasing. You can explain this algorithmically in plain English, but be clear.

**Problem 4.**

Explain clearly and completely why it follows that if a language and its complement are both Turing recognizable, then they are also Turing decidable.

**Problem 5.**

Describe how to design a deterministic Turing Machine that takes an encoding of a context-free grammar as its input, and decides whether or not its language is the set  $\{\epsilon\}$ , that is, the set that contains and only contains the empty string.